

Study on Distribution of First Natural Period (T_1) and Its Amplification Factor Derived from Response and Limit Strength Calculation for Subsurface Soil Layers

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The building code of Japan stand on performance-based type had been adopted since 2000. The recommended calculation procedure, Response and Limit Strength Calculation (RLSC), is adopting response spectrum method. The distribution of first natural period (T_1) and its amplification factor derived from Response and Limit Strength Calculation (RLSC) for subsurface soil layers is studied using simple subsurface soil model, up to three layers including engineering bedrock, considering several parameters such as thickness, soil types and shear wave velocity. The effectiveness of RLSC is examined comparing first natural period, T_1 , and its amplification factor, $G_s(T_1)$, derived from RLSC with those from SHAKE. When subsurface soil models are rather simple and be able to be replaced using equivalent one layer, T_1 and $G_s(T_1)$ results from RLSC are coincident with those from SHAKE within +/- 20% difference. Even though T_1 and $G_s(T_1)$ by RLSC show first natural period dependent trends, they are qualitatively explained considering conversion equation that connects amplitude in frequency and time domain. The differences of calculation way between SHAKE and RLSC are evaluated with respect to homogeneous soil models. The variation of displacements ratio RLSC by SHAKE distribute within +/-10%. Besides, the absolute value of displacements and process come to convergence show large variations in some cases. In order to increase accuracy and applicability of RLSC, the way of relative displacement calculation should be examined in detail.

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INTRODUCTION

The building code of Japan stand on performance-based type had been adopted since 2000. The basic concept for seismic design spectra consists of 1) basic design acceleration response spectra defined at the exposed engineering bedrock, and 2) evaluation of site response from geotechnical data of surface soil layers. In the procedure, iterative calculation is required in order to consider the strain-dependant soil deposit characteristics (nonlinear effect) of subsurface soil layer. The recommended calculation procedure, RLSC, is proposed in order to provide rather simple but can deal with nonlinear effect of subsurface soil layer semi-theoretically. RLSC is adopting response spectrum method and replaces the subsurface soil layers to a uniform stratum with an equivalent shear wave velocity [V_{se}], equivalent mass density [ρ_e] and equivalent damping coefficient [h_e], then evaluates amplification factor.

In this paper, first, we explain the procedures to evaluate acceleration response spectrum on ground surface by RLSC. Next, the distribution of first natural period (T_1) and its amplification factor derived from RLSC is investigated. The subsurface soil layer models up to three layers including engineering bedrock with several parameters such as thickness, soil types and shear wave velocity are adopted in analysis. The limits of these parameters are decided to satisfy actual ground condition. Finally, the applicability is examined comparing first natural period, T_1 , and its amplification factor, $G_S(T_1)$, derived from RLSC with those from SHAKE by Schnabel, et al. (1972) using same subsurface soil models.

ACCELERATION RESPONSE SPECTRUM AT GRAOUND SURFACE

Basic Response Spectrum at Engineering Bedrock

In RLSC, the earthquake load for evaluation is specified with earthquake ground motion. The evaluation earthquake ground motion is represented with the acceleration response spectrum in the following formula.

$$S_A(T) = ZG_S(T)S_0(T) \quad (1)$$

where, $S_A(T)$ is acceleration response spectrum for evaluation, Z is seismic zoning factor, $G_s(T)$ is soil amplification factor, $S_0(T)$ is the basic acceleration response spectrum at exposed (outcropping) engineering bedrock, and T is period in second. The engineering bedrock is defined as the layer with shear wave velocity larger than 400 m/s and certain thickness. The basic response spectrum has S_A uniform and S_V uniform parts as shown in Figure 1. The basic response spectra consist of different two levels, i.e., for life safety and damage limitation.

The level for life safety is based on the design force for the intermediate soil class specified in the prescriptive type of the provisions in Building Standard Law of Japan.

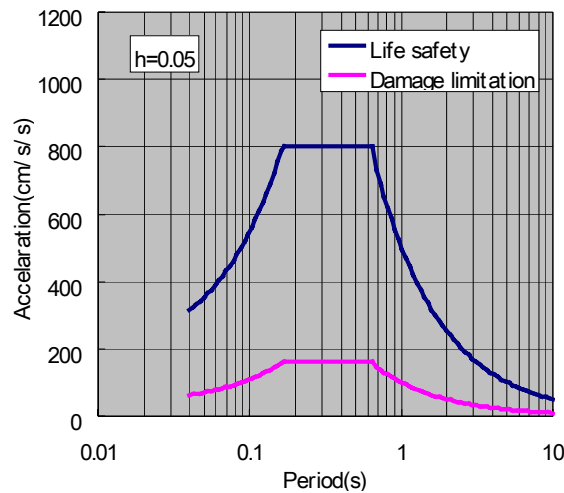


Figure 1 The basic response spectra for life safety and damage limitation

Evaluation Procedures of Acceleration Response Spectrum at Ground Surface

In order to evaluate the acceleration response spectrum on the ground surface at objective site, the amplification factor cause by subsurface soil deposits on the engineering bedrock is considered. The initial subsurface soil mode, i.e., the geotechnical data should be mostly obtained in the investigations conducted within the area. The recommended evaluation procedure RLSC is considering nonlinear soil properties as shown in Figure 2. The simplified analytical method, RLSC, is in accordance with the referring response spectrum method by Miura, et al.

(2001) with some modification, i.e., the Poisson's ratio is fixed 0.4 and Stodola method is adopted to calculate mode shape.

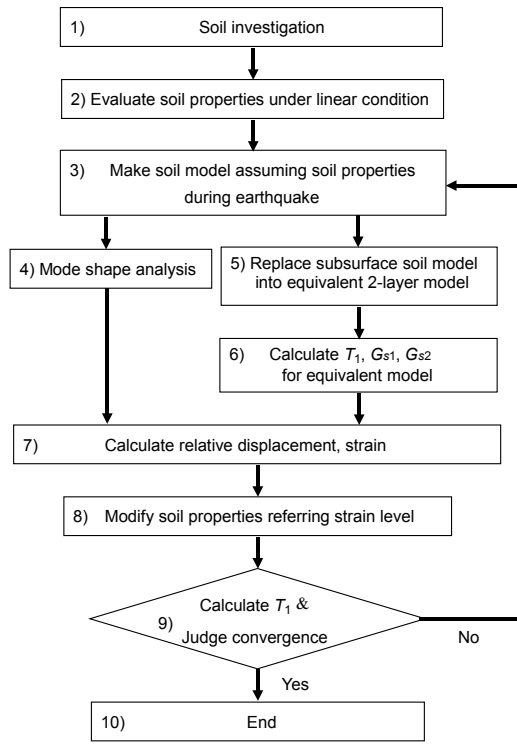


Figure 2 Procedures of iterative calculation

Transformation of response spectrum defined at outcropping engineering bedrock

The earthquake ground motion defined on the outcropping engineering bedrock is given as an acceleration response spectrum with 5% damping ratio $S_A(T, h = 0.05)$. A $S_A(T, h = 0)$, a velocity response spectrum $S_V(T, h = 0)$, and a Fourier spectrum of acceleration $F_A(T)$, have the approximate relations as

$$F_A(T) \approx S_V(T, h = 0) = (T / 2\pi)S_A(T, h = 0) \tag{2}$$

Mode shape analysis of soil profile

Subdividing the soil profile, a shear model of n-degrees of freedom is formed. The first mode shape vibration, U_i (normalized by the value at the surface) is obtained through the Stodola method. This mode shape is used to distribute displacement at surface to each subsurface soil layer.

Equivalent shear wave velocity and impedance

The subsurface soil layers at objective site are replaced to a uniform stratum with an equivalent shear wave velocity, V_{se} , an equivalent mass density, ρ_e , and an equivalent damping ratio, h_e , which are calculated using the properties of each soil layer.

$$V_{se} = \frac{\sum_{i=1}^{n-1} V_{si} H_i}{\sum_{i=1}^{n-1} H_i} \quad (3)$$

$$\rho_e = \frac{\sum_{i=1}^{n-1} \rho_i H_i}{\sum_{i=1}^{n-1} H_i} \quad (4)$$

where, V_{si} , H_i and ρ_i are shear wave velocity, layer height and mass density at the i -th layer from the surface, respectively. The shear wave velocity is defined $V_{si} = \sqrt{(G_i / \rho_i)}$ where G_i is shear modulus. These G_i reflect the nonlinear characteristics of the surface soil layers considering the $G - \gamma$ relationship of soil properties. The impedance of a wave motion, α , between the equivalent surface soil layer and the engineering bedrock is expressed as

$$\alpha = (\rho_i V_{se}) / (\rho_B V_{sB}) \quad (5)$$

where, V_{sB} and ρ_B are shear wave velocity and mass density at the engineering bedrock, respectively.

Amplification factor of subsurface soil layers

The amplification factor of the uniform subsurface soil layer to the outcropping engineering bedrock could be obtained by using the one-dimensional wave propagation in frequency domain. The amplification factor of the subsurface soil layers and the engineering bedrock to the outcropping one at first and second natural period are expressed as

$$G_s(T_1) = 1 / (1.57 h_e + \alpha) \quad (6)$$

$$G_s(T_2) = 1 / (4.71 h_e + \alpha) \quad (7)$$

for surface over outcropping engineering bedrock,

$$G_B(T_1) = 1.57 h_e / (1.57 h_e + \alpha) \quad (8)$$

for engineering bedrock over outcropping engineering bedrock. The equations (6) to (8) are defined following Osaki (1982).

The equivalent damping ratio, h_e , of each soil layer should reflect the nonlinear characteristics of the subsurface soil layers considering the $h - \gamma$ relationship.

Response acceleration and displacement of subsurface layers at the first natural period T_1

Adopting Eq. (2), the Fourier spectrum on the ground surface, $F_{SA}(T)$, is defined as

$$F_{SA}(T) = F_{0A}(T)G_S(T, h_e, \alpha_e) \approx (T/2\pi)S_{0A}(T, h=0)G_S(T, h_e, \alpha_e) \quad (9)$$

where, $G_S(T, h_e, \alpha_e)$ is amplification factor with equivalent h_e and α_e , $S_{0A}(T, h=0)$ is converted basic response spectrum with $h=0$. In RLSC, the response displacement at the first natural period on the ground surface, $D_S(T_1)$, and those at the lower boundary, $D_b(T_1)$, are defined as

$$\begin{aligned} D_S(T_1) &= (T_1/2\pi)^2 A_S(T_1) = (T_1/2\pi)^2 (1/T_1) F_{SA}(T) = (T_1/2\pi)^2 (1/T_1) F_{0A}(T) G_S(T, h_e, \alpha_e) \\ &\approx (T_1/2\pi)^3 (1/T_1) S_{0A}(T_1, h=0) G_S(T, h_e, \alpha_e) = \frac{T_1^2}{(2\pi)^3} \frac{1}{1.57h_e + \alpha_e} S_{0A}(T_1, h=0) \end{aligned} \quad (10)$$

$$D_b(T_1) \approx \frac{T_1^2}{(2\pi)^3} \frac{1.57h_e}{1.57h_e + \alpha_e} S_{0A}(T_1, h=0) \quad (11)$$

Displacements and uniform strain for each subsurface layers

The displacements at subsurface layers are defined distributing difference of $D_S(T_1)$ and $D_b(T_1)$ following the first mode shape U_i . The uniform strain for each subsurface layer is calculated dividing relative displacement by thickness for each of sublayers. Nonlinear relations between shear modulus G , damping factor h and shear uniform strain are modeled by the Romberg-Osgood.

ANALYTICAL METHOD

Considered models

The subsurface soil models considered in this study are simplified ones with several parameters, i.e., thickness, number, shear wave velocities, soil types of subsurface layers are selected as shown in Figure 3. Soil types are simply classified into clay or sand whose mass densities are 1.6, 1.8t/m³, respectively. As the models that have 4 times bigger V_1 than V_2 are not realistic and not adopted. The models composed of these parameters get up to 252 models. The initial damping coefficient of subsurface layers is fixed as 0.03. For engineering bedrock, shear wave velocity and the damping coefficient are also fixed as 400m/s and 0.02, respectively. In the analysis, subsurface layers are divided into 20 layers independent of whole thickness of subsurface layer.

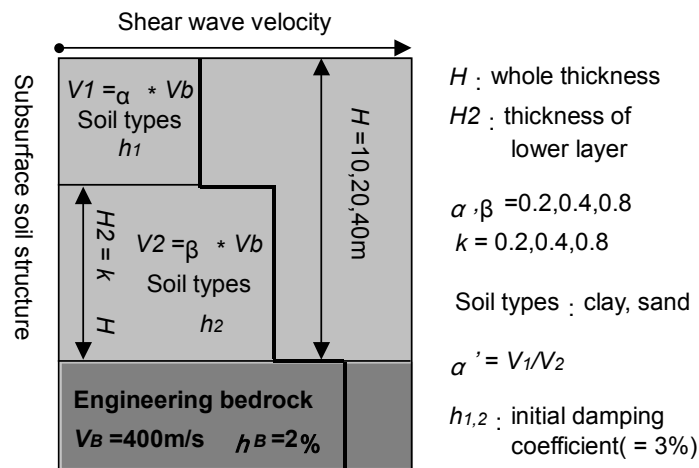


Figure 3 Considered parameters for soil models

Calculation methods

RLSC considers the strain-dependant soil deposit characteristics (nonlinear effect) of subsurface soil layers adopting iterative calculation in accordance with the former response spectrum method, Miura, et al. (2001). The judgment of convergence is satisfied if the variation of T_1 is less than 0.01.

In SHAKE, the 10 input seismic motions fitted basic response spectrum following “A Guideline for Composing Design Earthquake Ground Motion for Dynamic Analysis of Buildings” by BRI and BCJ (1992) were used.

Comparison

In RLSC, the amplification characteristic of subsurface layer, $G_s(T)$, is defined with first and second natural periods and two amplification factors, $G_s(T_1)$ and $G_s(T_2)$, at those natural periods. In SHAKE, the amplification characteristic is evaluated by the mean response spectral ratio by 10 input motions. Each of the response spectrum ratios was calculated dividing response spectrum from surface motion by one from input motion. Making a comparison, two types of values are examined, i.e., 1) first natural period and its amplification factor got from the spectral ratio, as comparison “type-A”, and 2) T_1 and $G_s(T_1)$ got following Notification No. 1457, the Ministry of Construction (2000), as comparison “type-B”.

RESULTS OF ANALYSIS

Figures 5 and 6 show amplification factor, $G_s(T)$, under damage limitation from RLSC and SHAKE, respectively. They are classified with homogeneity, α' , explained in Figure 3. In Figure 5, lower limit of amplification factor following Notification No. 1457, the Ministry of Construction (2000) was adopted. The predominance in 0.1 to 0.2 sec period range recognized in the figure of $\alpha'=0.25,0.50$ for SHAKE does not appear in RLSC. The amplification factors, $G_s(T)$, in RLSC are always equal or bigger than those of SHAKE. Figure 7 shows distribution of T_1 and $G_s(T_1)$ ratio, RLSC over SHAKE, under both of damage limitation and life safety for comparison “type-A”. The X-axis shows T_1 calculated by SHAKE. If T_1 and $G_s(T_1)$ by RLSC and SHAKE are coincident, T_1 and $G_s(T_1)$ ratios will be 1.0. In Figure 7, the lines correspond to +/- 20% differences against 1.0 are also depicted. Both of the situation under "damage limitation" and "life safety", the T_1 ratios decrease when T_1 get longer. The T_1 ratio under "damage limitation" result within 0.8 to 1.2 levels if α' is 0.5, 1.0 and homogeneous. Under “life safety”, the T_1 ratios get smaller than under "damage limitation" and lower than 1.0 except partial case for α' is 0.5. In case of the $G_s(T_1)$ ratios, they increase when T_1 get longer. The most of $G_s(T_1)$ ratios under "damage limitation" are distributed within 0.8 to 1.2 levels. If α' is 0.25 and T_1 is rather short, $G_s(T_1)$ can't be correctly evaluated because $G_s(T_1)$ ratios show

about 0.6. Under "life safety", $G_S(T_1)$ ratios show same tendency as under "damage limitation" with rather big values.

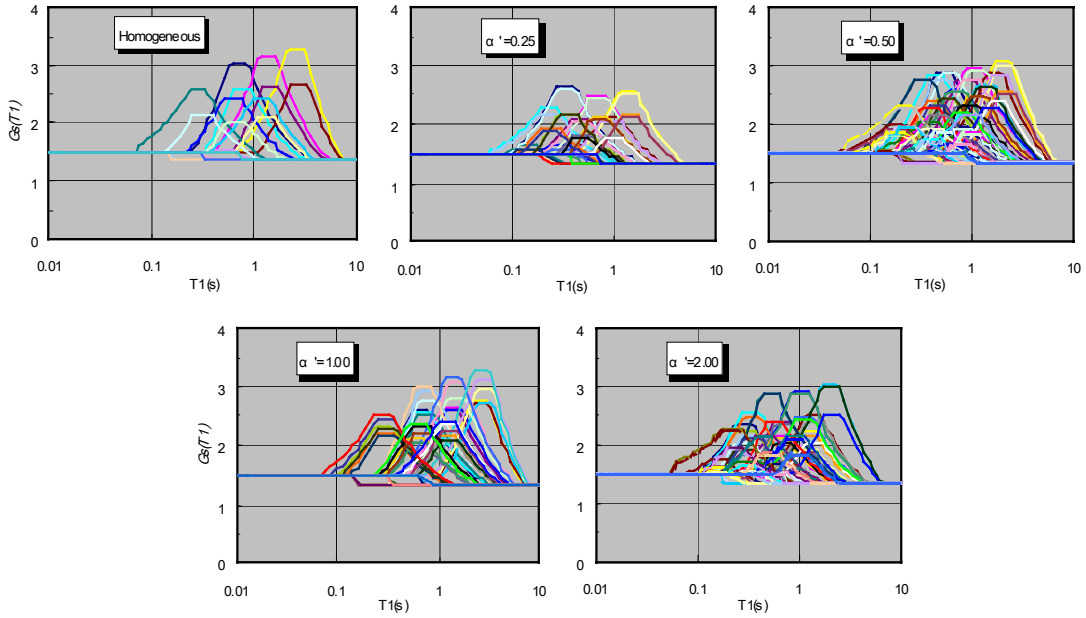


Figure 5 Amplification factor $G_S(T)$ derived from RLSC for damage limitation

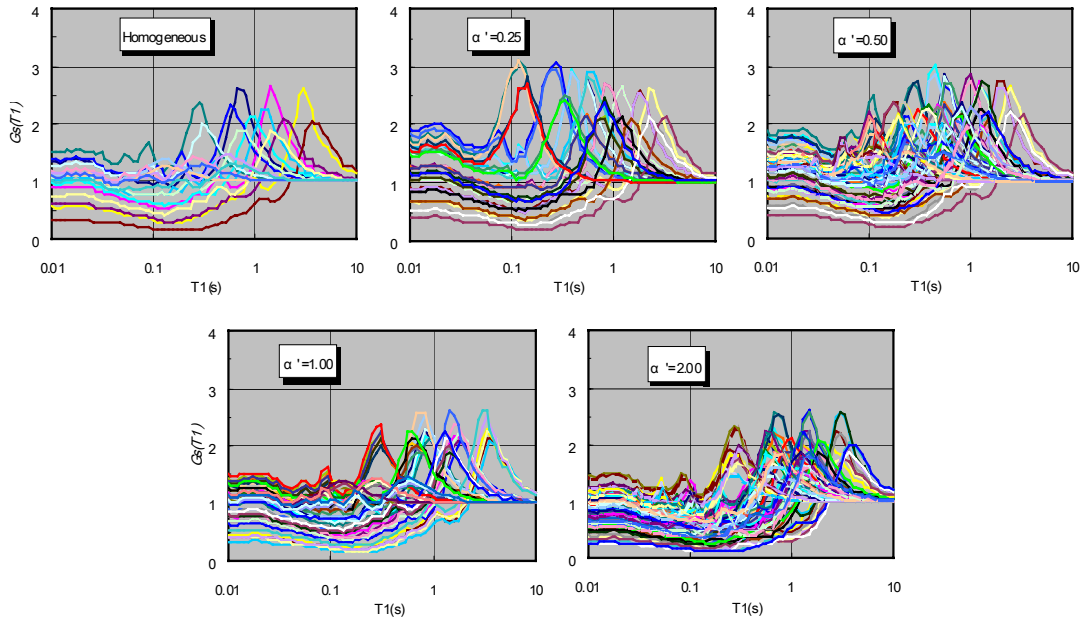


Figure 6 Amplification factor $G_S(T)$ derived from SHAKE for damage limitation

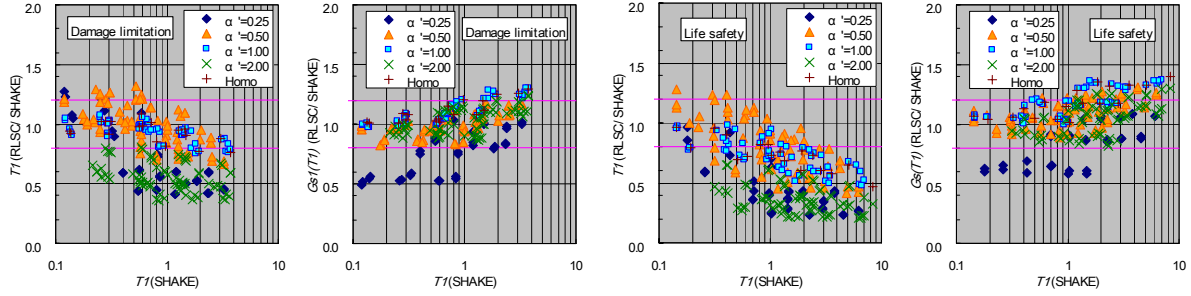


Figure 7 Distribution of T_1 and $G_S(T_1)$ ratio against T_1 by SHAKE for comparison “type-A”

The correlation coefficient was examined depending on α' value in order to indicate relation of T_1 calculated by RLSC and SHAKE with values. As shown in Table 1, about T_1 , they get bad under “life safety” than under "damage limitation" in all cases. The correlation coefficients for $G_S(T_1)$ show the almost same values under both of damage limitation and life safety except in case that α' is 0.25.

Table 1 Cross correlation coefficient with α' for “type-A”

α'		Homogeneous	0.25	0.50	1.00	2.00
Damage limitation	T_1	0.993	0.955	0.967	0.995	0.935
	$G_S(T_1)$	0.974	-0.069	0.901	0.972	0.937
Life safety	T_1	0.978	0.918	0.928	0.977	0.890
	$G_S(T_1)$	0.962	-0.194	0.805	0.976	0.934

Figure 8 shows distribution of T_1 and $G_S(T_1)$ ratio following comparison “type-B” evaluation explained at “Comparison”. In this evaluation type, the distribution of T_1 and $G_S(T_1)$ show the same but not so strong tendency as “type-A”. The T_1 and $G_S(T_1)$ ratios result within 0.73 to 1.28 and 0.88 to 1.21 levels, respectively for all analytical cases. In Table 2, the correlation coefficient was examined depending on α' value in order to indicate relation of T_1 as same as in “type-A”. All of the correlation coefficients are improved than in “type-A” and over 0.948. And that they show the same values in “damage limitation” and “life safety” independent of α' , T_1 and $G_S(T_1)$.

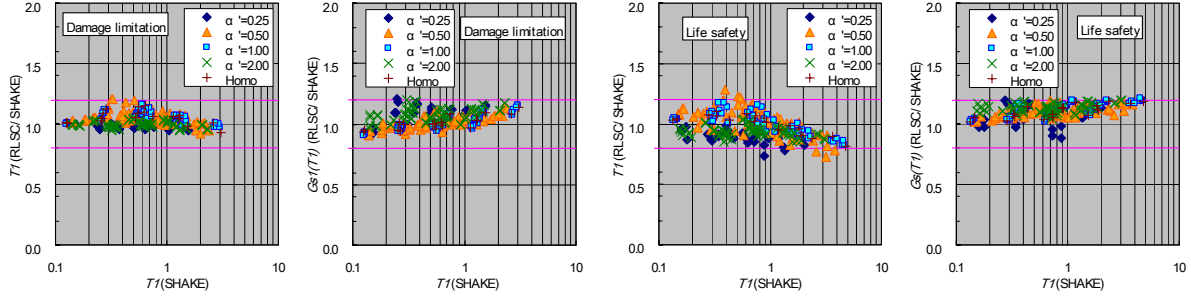


Figure 8 Distribution of T_1 and $G_S(T_1)$ ratio against T_1 by SHAKE for comparison “type-B”

Table 2 Cross correlation coefficient with α' for “type-B”

		α'	Homogeneous	0.25	0.50	1.00	2.00
Damage limitation	T_1		0.997	1.000	0.997	0.999	0.999
	$G_S(T_1)$		0.981	0.948	0.966	0.979	0.972
Life safety	T_1		0.997	0.995	0.988	0.997	0.998
	$G_S(T_1)$		0.988	0.953	0.978	0.993	0.991

In summarize, the first natural period, T_1 , gets good results when α' is 0.50, 1.0 and homogeneous under "damage limitation". About $G_S(T_1)$, the correlation coefficient doesn't depend on input motion level, i.e., "damage limitation" and "life safety", shows good relation when α' is 1.00, 2.00 and homogeneous. The correlation coefficients from “type-B” values are higher than those from “type-A”. As the amplification factor was set uniform from T_1 -20% to T_1 +20% period range with $G_S(T_1)$ level by Notification No. 1457, the Ministry of Construction (2000) relate to RLSC, the 20% difference against SHAKE was considered to some extent.

Nevertheless, first natural period dependant trends could be seen. Then, the equivalent shear wave velocities, V_{se} , and the equivalent damping ratios, h_e , that characterizes calculated T_1 and $G_S(T_1)$, by RLSC and SHAKE under converged state were compared. As shown in Figure 9, both V_{se} and h_e ratio also have first natural period dependent trend, i.e., V_{se} ratio increase and h_e decrease together with T_1 . In Eq.10, Fourier amplitude was divided by first natural period, T_1 , to get time domain amplitude about T_1 component. This operation decreases time domain amplitude in accordance with increase of period. For instance, assuming that the amplitude about two arbitrary periods T_x and $2T_x$ in frequency domain are equal, i.e. $F_{SA}(T_0)$ is equal to $F_{SA}(2T_0)$, the converted amplitude into time domain will be $(1/T_x)F_{SA}(T_x)$ and $(1/2T_x)F_{SA}(2T_x)$,

i.e., the amplitude in time domain for period T_x is double than that for period $2T_x$ because T_x is the half of $2T_x$. When the amplitude in time domain decreases, the strain-dependant characteristics of soil does not progress, and then V_{se} doesn't decrease and h_e doesn't increase. The trend appeared in Figure 9 could be explained qualitatively.

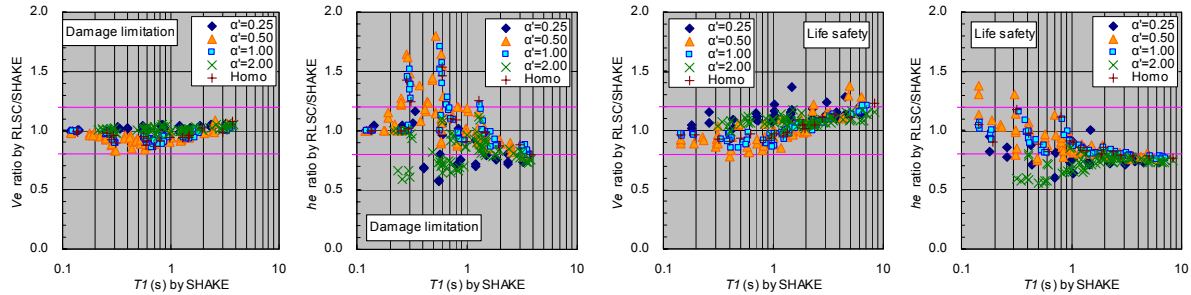


Figure 9 Distribution of V_{se} and h_e ratio against T_1 by SHAKE under damage limitation and life safety

As the way of calculation at steps 1) to 3), 8) and 9) in Figure 2 are quite same and the steps 4) to 7) are applying different ways by SHAKE and RLSC, it's consider that these variations mainly cause by steps 4) to 7). Then, the characteristics of steps 4) and 7) were evaluated. In order to simplify, only the case of homogeneous soil models that showed rather good coincidence with SHAKE were considered. The parameters for homogeneous soil model are shown in Table 3. The models composed of these parameters are 18.

The examples of mode shape normalized by displacement at surface with respect to 10, 20 and 40 m thickness of subsurface layer models under initial soil conditions are depicted in Figure 10. The shear wave velocity and soil type of subsurface layer are 80m/s and clay, respectively. In case of SHAKE, the relative displacements were calculated multiplying uniform strain by thickness for each of subsurface layers and summed up to get displacement at surface. In case of RLSC, displacement at surface and bedrock were calculated through equation (10) and (11). The mode shapes by SHAKE and RLSC are coincident well independent of analytical way, soil type and thickness. For all homogeneous models under initial soil conditions, the variation of displacements ratio RLSC by SHAKE has different shape by shear wave velocity and thickness but distribute within +/-10% as shown in Figure 11. However, the absolute value of displacements and process come to convergence show large variations in some cases. In SHAKE, the displacements at surface increase and get convergence through iteration. Besides in RLSC, the displacements at 1st iteration are not always minimum value, some cases took middle or maximum values, and got convergence. Even in the converged displacement by RLSC shows

large difference compared with SHAKE. In some cases, it gets up to more than double. For example, Figure 12 shows distribution of displacements until come to convergence. The displacements at surface change 4 cm to 7.5 cm by SHAKE and 18 cm to 11.5 cm by RLSC.

Table 3 Parameters for homogeneous soil model

	Thickness (m)	Vs (m/s)	Soil types	Initial Damping (h)
Homogeneous layer	10/20/40	80/16/320	Clay/sand	0.03
Engineering bedrock	----	400	----	0.02

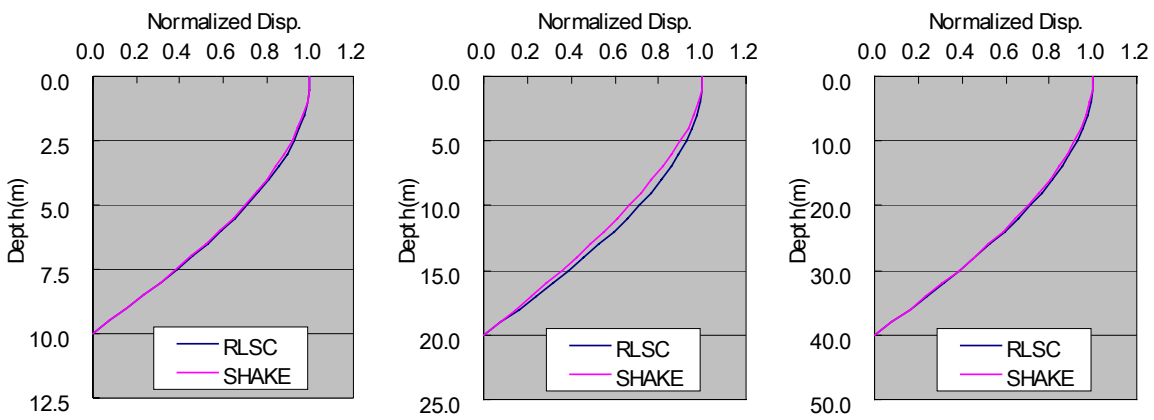


Figure 10 Examples of mode shape by SHAKE and RLSC with respect to 10, 20 and 40 m thickness of subsurface layer models. The shear wave velocity and soil type of subsurface layer are 80m/s and clay, respectively.

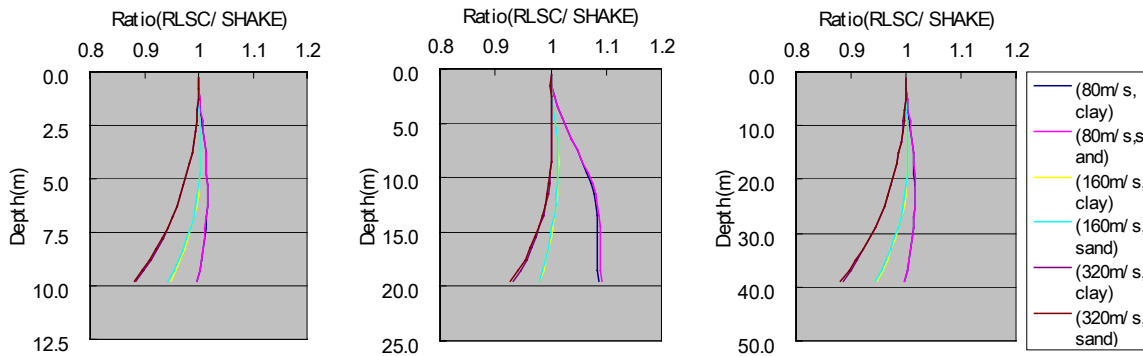


Figure 11 Relative displacement ratio (RLSC/SHAKE) with respect to depth.

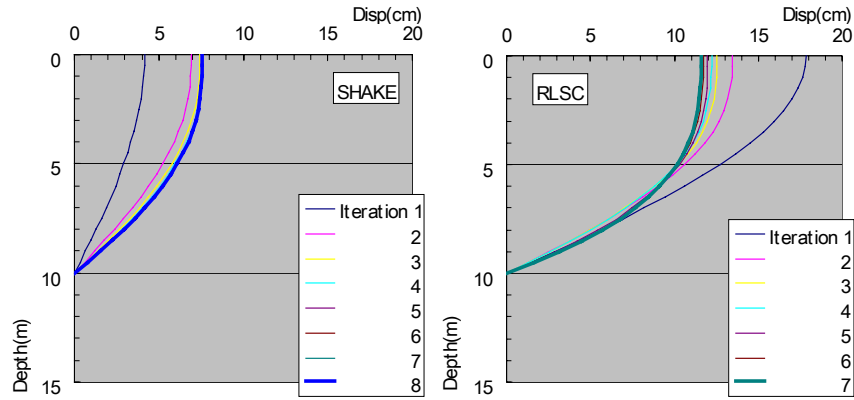


Figure 12 Distribution of displacements until come to convergence with respect to iteration number. The shear wave velocity, soil type of subsurface layer and thickness of subsurface layer models are 80m/s, clay and 10m, respectively.

CONCLUSIONS

The applicability of Response and Limit Strength Calculation (RLSC) for subsurface soil layers was studied. One layer subsurface models and simple two layers models similar to homogeneous layer showed that both of T_1 and $G_s(T_1)$ derived from RLSC and SHAKE were usually coincident each other. In case that the subsurface models consisted of complicated two layers and hard to be replaced by an equivalent uniform stratum model, T_1 and/or $G_s(T_1)$ from RLSC were different from those of from SHAKE with more than 20% difference, in some cases over 100% variation. As the amplification factor is set uniform from T_1 -20% to T_1 +20% period range with $G_s(T_1)$ level by Notification No. 1457, the Ministry of Construction (2000) relate to RLSC, the 20% difference against SHAKE is considered to some extent. While, T_1 and $G_s(T_1)$ by RLSC showed first natural period dependent trends. They were also recognized in the equivalent shear wave velocities, V_{se} , and the equivalent damping ratios, h_e , that characterizes calculated T_1 and $G_s(T_1)$. Considering conversion equation that connects amplitude in frequency and time domain, they are qualitatively explained. The differences of calculation ways between SHAKE and RLSC, the characteristics of mode shape analysis and relative displacement calculation, were evaluated with respect to homogeneous soil models. The variation of displacements ratio RLSC by SHAKE had different shape by shear wave velocity and thickness but distributed within +/- 10%. The absolute value of displacements and process come to convergence showed large variations in some cases. In order to increase accuracy and applicability of RLSC, the way of

relative displacement calculation should be examined in detail. The effects by Replacement of subsurface soil model into equivalent 2-layer model and calculation of T_1 , Gs_1 , Gs_2 for equivalent model, not examined in this paper, also should be examined.

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REFERENCES

- BRI and BCJ, 1992. "A Guideline for Composing Design Earthquake Ground Motion for Dynamic Analysis of Buildings", Appendix of Annual Report of the Committee on Technical Guideline for Design Earthquake Ground Motion, (in Japanese).
- Miura, K., Koyamada, K. and Iiba, M., 2001. "Response spectrum method for evaluating nonlinear amplification of surface strata", Journal of Structural and Construction Engineering, Transaction of Architectural Institute of Japan, No.539, 57-62, AIJ
- Ohsaki Y., 1982. "Dynamic characteristics and one-dimensional linear amplification theories of soil deposits", Research Report 82-01, Dept. of Architecture, Faculty of Eng., Univ. of Tokyo, March.
- Schnabel, P. B., Lysmer, J. and Seed, H. B., 1972. "SHAKE - A Computer Program for Earthquake Response Analysis of Horizontally Layered Sites", Report No. EERC 72-12, EERC.